Construction of sub-Poissonian radiation fields

1979 J. Phys. A: Math. Gen. 12 L35

(http://iopscience.iop.org/0305-4470/12/1/009)

J. Phys. A: Math. Gen., Vol. 12, No. 1, 1979. Printed in Great Britain

LETTER TO THE EDITOR

Construction of sub-Poissonian radiation fields†

H P Baltes‡, A Quattropani§ and P Schwendimann||

‡ Zentrale Forschung und Entwicklung, Landis & Gyr Zug, CH-6301 Zug, Switzerland
§ Laboratoire de Physique Théorique, Ecole Polytechnique Fédérale Lausanne, CH-1006 Lausanne, Switzerland
|| Institut für Theoretische Physik, Universität Bern, CH-3012 Bern, Switzerland

Received 12 October 1978

Abstract. Two classes of states showing sub-Poissonian photon statistics are constructed by appropriate mixtures of n-photon states. These mixtures are possibly relevant for antibunching.

It is well known that coherent and incoherent radiation can be distinguished by their statistical properties. Coherent fields are characterised by Poissonian statistics, whereas chaotic fields show geometric photon distribution (Arecchi 1969, Pike 1969). For such distributions the second factorial moment obeys

$$\langle a^+a(a^+a-1)\rangle - \langle a^+a\rangle^2 \geqslant 0 \tag{1}$$

where equality corresponds to the Poissonian case, a^+ and a denote the single-mode amplitude operators, and the angular brackets denote statistical averages. In this letter we look for states which obey

$$\langle a^+a(a^+a-1)\rangle - \langle a^+a\rangle^2 < 0. \tag{2}$$

These states may be called 'sub-Poissonian'. States with sub-Poissonian fluctuations (2) are involved in several different phenomena:

(a) Antibunching occurring in degenerate parametric amplification (Stoler 1974, Mišta and Periña 1977), two-photon absorption (Simaan and Loudon 1975, Hildred and Hall 1978), and the resonant Stark effect (Carmichael and Walls 1976, Carmichael *et al* 1978 and references therein).

(b) Superradiant emission of two-level atoms excited by a coherent pulse (Bonifacio *et al* 1971).

(c) Moreover, such low-noise fields could be relevant as a source for optical communication, since the capacity of any information channel is known to be limited by noise (Shannon 1948).

The purpose of this letter is to construct radiative states satisfying condition (2) without considering specific physical models. To this end we rewrite (2) in the form

$$[\langle a^+a(a^+a-1)\rangle - \langle a^+a\rangle^2]/\langle a^+a\rangle^2 = -A, \qquad A>0 \tag{3}$$

where the positive constant A measures the relative deviation from the Poissonian case. We express condition (3) in terms of two standard generating functions, namely

† Presented at the Optics '78 Conference held at the University of Bath, 20–23 September 1978.

that of the factorial moments,

$$Q_1(x) \equiv \sum_{\nu=0}^{\infty} (-x)^\nu \langle (a^+)^\nu a^\nu \rangle / \nu! \tag{4}$$

and that of the cumulants,

$$Q_2(x) \equiv \ln Q_1(x). \tag{5}$$

In terms of $Q_1(x)$, relation (3) reads

$$\left[\frac{d^2 Q_1(x)}{dx^2} - (1-A)\left(\frac{dQ_1(x)}{dx}\right)^2\right]_{x=0} = 0. \tag{6}$$

In terms of $Q_2(x)$, however, relation (3) takes the form

$$\left[\frac{d^2 Q_2(x)}{dx^2} + A\left(\frac{dQ_2(x)}{dx}\right)^2\right]_{x=0} = 0. \tag{7}$$

Since equations (6) and (7) only involve first and second moments of the field, subsidiary conditions have to be introduced in order to define the state. This may be achieved by extending equations (6) and (7) to the interval $0 \leqslant x \leqslant 1$. This extension imposes conditions on the higher factorial moments and cumulants, respectively. In this way, we obtain two differential equations defining two classes of sub-Poissonian states. The definitions (4) and (5) imply

$$Q_1(0) = 1, \qquad Q_2(0) = 0 \tag{8}$$

(normalization) and

$$dQ_1/dx|_{x=0} = dQ_2/dx|_{x=0} = -\langle a^+ a \rangle \equiv -n \tag{9}$$

where n denotes the average field intensity. We use the relations (8) and (9) as initial conditions for the differential equations.

We first discuss the solution of the differential equation obtained from equation (7), which reads

$$Q_2(x) = A^{-1} \ln(1 - nAx), \qquad 0 \leqslant x \leqslant 1. \tag{10}$$

The logarithmic function occurring in (10) is defined in the interval $0 \leqslant x \leqslant 1$ only if we require $nA \leqslant 1$. Thus very large relative deviations from Poissonian statistics are possible only for weak intensities. The factorial moments of the state defined by $Q_2(x)$ are easily derived from equations (4), (5) and (10), and read

$$\langle (a^+)^\nu a^\nu \rangle = (nA)^\nu \Gamma(1 + A^{-1}) / \Gamma(1 - \nu + A^{-1}) \tag{11}$$

with $\nu = 0, 1, 2, \ldots$. We notice that the fluctuations are sub-Poissonian for any constant A obeying $0 < A \leqslant n^{-1}$. The case $A = n^{-1}$ corresponds to the pure n-photon state. In the limit $A \to 0$ the moments of the coherent state are reproduced. Thus we have shown that mixed states with sub-Poissonian statistics exist.

We now discuss the solution of the differential equation obtained from condition (6), which reads

$$Q_1(x) = (A-1)^{-1} \ln[(1-A)nx + 1] + 1, \qquad 0 \leqslant x \leqslant 1. \tag{12}$$

Again we have to require $nA < 1$. The corresponding factorial moments are given by

$$\langle (a^+)^\nu a^\nu \rangle = (\nu - 1)! n^\nu (1-A)^{\nu-1}, \qquad \nu \geqslant 1. \tag{13}$$

Sub-Poissonian fluctuations are again obtained for $0 < A \leq n^{-1}$. In contrast to (11), we notice that now also the case $A = n^{-1}$ corresponds to a mixed sub-Poissonian state. In the limit $A \to 0$ we now obtain a new kind of state, whose second moment obeys

$$\langle (a^+)^2 a^2 \rangle = \langle a^+ a \rangle^2 = n^2. \tag{14}$$

Thus the second moment of this state factorises, whereas the higher moments

$$\langle (a^+)^\nu a^\nu \rangle = (\nu - 1)! n^\nu \tag{15}$$

apparently do not. This particular state may be called 'sub-coherent', since its statistical behaviour is intermediate between the chaotic and the coherent state; the sub-coherent state exhibits coherence of exactly the second order. It is of some theoretical interest as an implementation of Glauber's (1969) definition of nth-order coherence. To the best of our knowledge, states exhibiting some exact order of coherence between 1 and ∞ have not been constructed hitherto. However, the state characterised by (15) is perhaps not of much practical interest since the average intensity has to be too small, namely $n \leq 1$.

We remark that the above two classes of sub-Poissonian mixed states are constructed under special restrictions imposed on the higher moments More general construction schemes, such as maximisation of the entropy or similar functions, do not seem to lead to tangible results.

References

Arecchi F T 1969 *Quantum Optics* ed. S Kay and A Maitland (New York: Academic) p 57
Bonifacio R, Schwendimann P and Haake F 1971 *Phys. Rev.* A **4** 854
Carmichael H J, Drummond P, Meystre P and Walls D F 1978 *J. Phys. A: Math. Gen.* **11** L121
Carmichael H J and Walls D F 1976 *J. Phys. B: Atom. Molec. Phys.* **9** L43
Glauber R J 1969 *Quantum Optics* ed. S Kay and A Maitland (New York: Academic) p 15
Hildred G P and Hall A G 1978 *J. Phys. A: Math. Gen.* **11** L209
Mišta L and Peřina J 1977 *Acta Phys. Polon.* A **52** 425
Pike E R 1969 *Quantum Optics* ed. S Kay and A Maitland (New York: Academic) p 160
Shannon C E 1948 *Bell Syst. Tech. J.* **28** 397
Simaan H D and Loudon R 1975 *J. Phys. A: Math. Gen.* **8** 539
Stoler D 1974 *Phys. Rev. Lett.* **33** 1397